into the swirl chamber, and consequently on the vortex circulation, therefore, is also smaller. Indeed, in the experiment of Kraus and Cutler[3] using similar nozzle geometries, measured mass flow rates and rates calculated based on plenum conditions using the present method agreed within 8%.

Conclusion

A simple analytical method for the design of convergent–divergent nozzles that produce axisymmetric vortex flows has been presented. The method assumes compressible, isentropic, quasi-one-dimensional flow. Experimental results confirm the applicability of the method and give an indication of the errors due to viscous effects at the core.

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Structural Damage Detection Using the Simulated Evolution Method

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Introduction

SYSTEM identification methods have been extensively employed in recent years for structural damage assessment.[1,2] The theoretical background of system identification methods is that the departure of the measured response from analytically predicted values can be used to identify changes in system parameters such as elements of stiffness matrix, which in turn reflect possible structural damage. Most system identification methods, which are formulated by the output-error method or the equation-error method, essentially result in unconstrained or constrained optimization problems.[3] The major problems associated with such optimization-based approaches include the loss of physical connectivity associated with a model and the overparameterization of an analytical model, which may result in the problem of identifiability, that is, lack of uniqueness in the identified parameter values.

Berman[4,5] pointed out that there is no unique valid linear discrete model to be identified using limited data obtained from a dynamic test. To alleviate the problem of identifiability, a two-stage damage detection approach has been proposed.[6,7] In the first stage, the residual force method is employed to locate candidate damage parts in a structural system; in the second stage, the damage extent of the candidate damage parts is then evaluated using an optimization method. The residual force method is effective in damage localization using insuffucient modal data as long as the modeling error is small for the undamaged structure.[7,8] Once the location of possible damage in a structural system is identified, the damage extent is further evaluated using an optimization method. In the present study, we employ the method of simulated evolution, which is a global optimization method and has the capability of locating multiple global optima within a specified domain.[9,10] By using the global optimization method, we can ensure that some or all of the possible damage scenerios, if theoretically nonunique results do exist, can be found in the second stage for further assessment.

Two-Stage Damage Detection Method

To effectively reduce the number of parameters to be identified and to alleviate the problem of lack of uniqueness in the result of structural damage analysis, a two-stage structural damage detection method is employed.

In the first stage of damage localization, the aim is to locate possible damage areas so that the damage extent can be estimated more reliably in a separate stage. In this study, the residual force method[7,8] is employed. By assuming that the mass matrix $\boldsymbol{M}$ is unchanged as damage occurs, we introduce the relation

$$\boldsymbol{K}_d = \boldsymbol{K}_0 + \Delta \boldsymbol{K} \tag{1}$$

where $\boldsymbol{K}_d$ and $\boldsymbol{K}_0$ are the stiffness matrices associated with the damaged and undamaged structural models, respectively, and $\Delta\boldsymbol{K}$ is the corresponding changes. Define the residual force vector $\boldsymbol{R}_i$ of the ith mode as

$$\boldsymbol{R}_i \equiv \left(\boldsymbol{K}_0 - \omega_d^2 \boldsymbol{M}\right)\phi_{di} = -\Delta\boldsymbol{K}\phi_{di} \tag{2}$$

where ω_{di} and ϕ_{di} are, respectively, the ith modal frequencies and mode shapes of the structure after damage occurs. The second equality in Eq. (2) follows from Eq. (1) and the eigenequations associated with the structural system after damage. By inspecting Eq. (2) it can be seen that the residual force vector $\boldsymbol{R}_i$ will have only nonzero elements at the degrees of freedom (DOFs) affected by damage (nonzero elements in $\Delta\boldsymbol{K}$). It is remarkable that the residual force vector $\boldsymbol{R}_i$ can be determined directly using the known analytical model (must be accurate enough) and the measured postdamage eigendata for any mode available, as shown in Eq. (2).

In the second stage, the candidate damaged elements (or substructures) are further evaluated to determine their damage extent. To reduce effectively the number of parameters while retaining physical connectivity of the model, we introduce reparameterization of the stiffness matrix as follows:

$$\boldsymbol{K}_d = \boldsymbol{K}_0 + \Delta\boldsymbol{K} = \boldsymbol{K}_0 - \sum_{i=1}^{N_d} \theta_i \boldsymbol{K}_i \tag{3}$$

where N_d is the number of possibly damaged elements (or substructures) that are identified in the first stage and θ_i, $i = 1 \sim N_d$, are the corresponding damage parameters. The value of θ_i is 1 if the ith element is completely damaged and $\theta_i = 0$ if the ith element is not damaged at all.

To identify the damage parameters associated with those elements that are possibly damaged, the least-squares output-error method[10] is employed. In this study, the error criterion function is defined through the modal information as

$$J_2(\boldsymbol{\theta}) = \sum_{i=1}^{M} \left\{ \left(\frac{\omega_i - \omega_{di}}{\omega_{di}} \right)^2 + [1 - \mathrm{MAC}(\phi_i, \phi_{di})] \right\} \tag{4}$$

In Eq. (4), ω_i and ω_{di} are the ith natural frequencies of the model and of the damaged system, respectively. Modal assurance criterion (MAC) is a measure of correlation between two sets of mode shapes and is defined as follows[11]:

$$\mathrm{MAC}(\phi_i, \phi_{di}) = \frac{\left|\phi_i^T \phi_{di}^*\right|}{\phi_i^T \phi_i^* \phi_{di}^T \phi_{di}^*} \tag{5}$$

where T denotes matrix transpose and * denotes complex conjugate. By minimizing the error function $J(\boldsymbol{\theta})$ with respect to the damage parameters $\boldsymbol{\theta}$, the extent of damage for those elements that are possibly damaged can be determined.

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It is remarkable that conventional optimization methods, such as the gradient methods, are easily trapped into local minima for the damage detection problem, in which the hypersurface defined by the error function in the parameter space is highly nonlinear with multiple local (or even global) minima. To circumvent this problem, the method of simulated evolution, which has been shown to have the capability of effectively locating global minima for function optimization,[9,10] is employed.

Method of Simulated Evolution

The method of simulated evolution is a global optimization method based on the Darwinian evolutionary theory. Fogel and Atmar[12] studied the evolution mechanism based on simulated sexual recombination. Their results indicated that modifying each component of the evolving solution by a Gaussian random variable results in an efficient search for global optima. The capability of the evolutionary search method in locating the global minimum among numerous local minima has been demonstrated and has been applied for identification of modal parameters of linear vibrating structures.[10]

The method of simulated evolution simulates the natural evolution process, which consists of four major steps: 1) reproduction, 2) mutation, 3) competition, and 4) selection. The evolutionary search is implemented as follows[10]:

1) An initial population of trial vectors $\boldsymbol{p}_i$, $i = 1, \ldots, K$, is chosen at random from a uniform distribution ranging over $[a, b]$ in n dimensions, where n is the number of variables.

2) Each $\boldsymbol{p}_i$ produces an offspring vector $\boldsymbol{p}_{i+K}$ with Gaussian random replicative error. Denoting $\boldsymbol{p}_{i,j}$ as the jth element of the ith organism, we define $\boldsymbol{p}_{i+K,j} = \boldsymbol{p}_{i,j} + N(0, \sigma^2)$, $\forall j = 1, \ldots, n$, where $N(\mu, \sigma^2)$ represents a Gaussian random variable with mean μ and variance σ^2.

3) Each of the existing $2K$ organisms undergoes competitions (function value comparisons) with m randomly chosen organisms.

4) The first K organisms having the most wins survive as parents for the next generation, and the remaining K organisms with fewer wins will be eliminated.

5) The process repeats by returning to step 3. A halt to the sequence is declared when either a predetermined quality of solution has been achieved or a set number of iterations have been exhausted.

The method of simulated evolution has the general advantage of locating global optima without the need of making an initial estimate of parameter values. Thus, it is suitable for structural damage detection analysis in which damage in a structure is to be identified from the limited data, for example, incomplete modes, available in practice.

Numerical Studies

A plane truss structure containing 13 elements is simulated as shown in Fig. 1, where the boldfaced numbers indicate the node number and the numbers in parentheses indicate the element number. The DOFs that are affected by each element of the truss structure are listed in Table 1 for easy reference. In this example, we assume that the analytical model can well represent the structure within the range of the first 4 modes (out of a total of 13 modes), so that the effect of modeling error on the residual forces of the first four modes can be neglected.

In the simulation, we consider the case where elements 2, 7, and 12 are all damaged with 50% reduction in stiffness. In the first stage of the analysis, we use the method of residual force to locate the elements that are possibly damaged. The residual force vectors of the first two modes are shown in Figs. 2a and 2b, respectively. It is obvious that residual forces with nonzero values show up at the

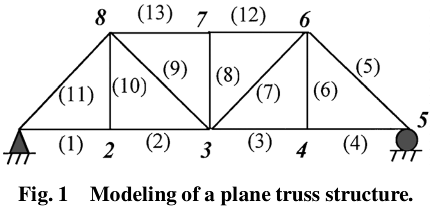

Fig. 1 Modeling of a plane truss structure.

Table 1 DOFs that are related to each element in the truss

Element number	DOF(s) affected
1	1
2	1, 3
3	3, 5
4	5, 7
5	7, 8, 9
6	6, 9
7	3, 4, 8, 9
8	4, 11
9	3, 4, 12, 13
10	2, 13
11	12, 13
12	8, 10
13	10, 12

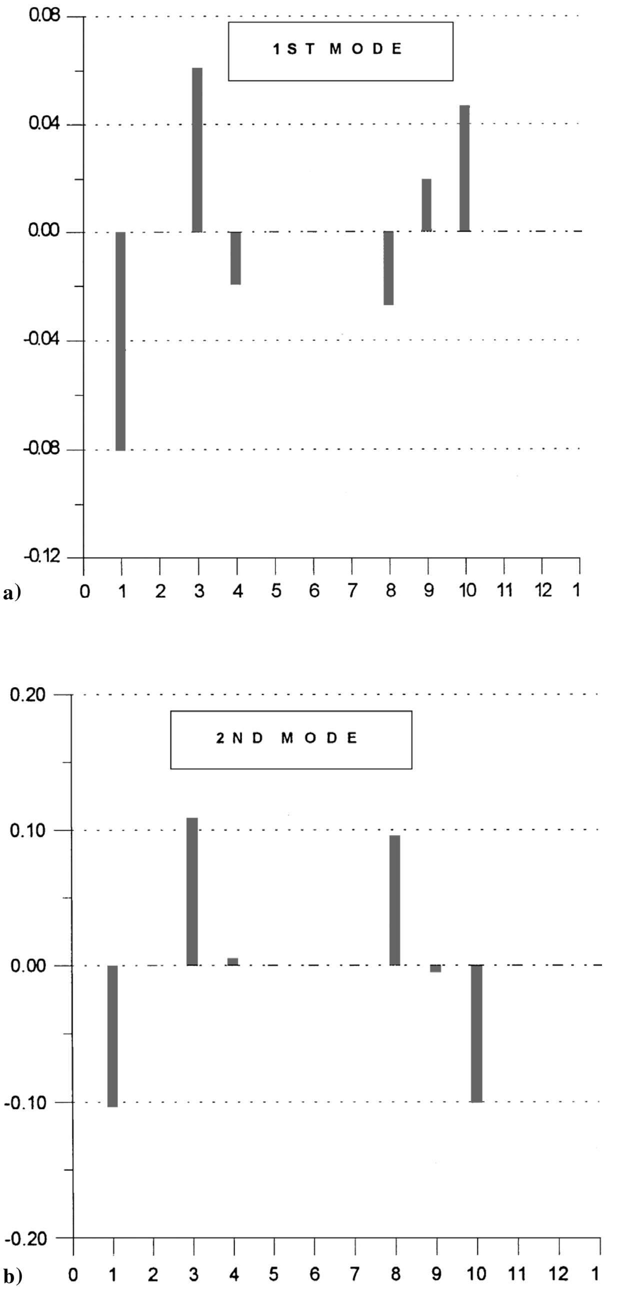

Fig. 2 Residual force vectors of the first two modes for the truss structure.

DOFs numbered 1, 3, 4, 8, 9, and 10. According to the relation between the element number and the DOF number (Table 1), we can find that damage could have occurred in the elements numbered 1, 2, 5, 7, and 12.

In the second stage of analysis, the damage extents of the five candidate elements are determined using the output-error approach in which the error criterion function to be minimized is defined as given by Eq. (4) using the first four modes. By using the method of simulated evolution for optimization, we obtain the damage parameters θ_i, $i = 1$–5, associated with each of the five elements, which are 4.93, 47.36, 0.00, 50.01, and 50.14(%), respectively, in 1000 generations of evolution with a population size of 50. It is fair to assume that damage occurs only in the three elements numbered 2, 7, and 12. When performing the optimization process once again using only three damage parameters, we obtain the exact results as simulated. It is remarkable that, because we used the method of simulated evolution in the process of optimization, an initial estimate on the values of the damage parameters is not needed and the result is reliable even in the case where multiple optima exist (if fewer modes are employed).

Conclusions

A two-stage structural damage detection method is proposed in which the residual force method and the method of simulated evolution are employed, respectively, in different stages to alleviate the problem of lack of identifiabilty in finding variations in the stiffness matrix of a structure subject to damage. The damage localization algorithm based on the residual force method was shown to successfully locate structural damage in the case that the analytical model used for damage detection can well represent the dynamical system of interest within the (frequency) range of measurement (eigenmodes) data employed. The method of simulated evolution was also shown to be effective in accurately evaluating the damage extents of the candidate elements that have been identified in the first stage. Further studies are merited on the effect of incomplete modal data with model reduction and on the applicability of this damage detection technique to real-life problems involving more complex structures.

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